

A NEW APPROACH TO LOCALITY AND CAUSALITY⁺

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Abstract

In the light of some recent results, it is argued that usual concepts of causality and locality are approximations valid at scales greater than the Compton wavelength and corresponding time scales. It follows that the "spooky" non-locality of Quantum Mechanics is not really so and in fact is perfectly consistent with a recently discussed holistic model, which again is corroborated by latest astrophysical and cosmological observations. This approach also provides a rationale for the origin of the metric and points to, what may be called a space time quantization which may be, ultimately, fundamental.

1 Introduction

In classical physics, causality has broadly three meanings[1]. 1) Causality as predictability, what may be called Newtonian causality. 2) The fact that no signal can have a superluminal velocity. 3) The fact that advanced effects of fields with a finite propagation velocity are forbidden (that is the future cannot affect the present or the past).

Quantum Mechanics retains the tenets of special relativity, but we speak of microscopic causality[2]: 4) The fact that observables separated by a space like

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interval can be simultaneously measured, more specifically they commute. However it must be borne in mind that the space time of classical physics is not only deterministic, but it is also meaningful to speak in terms of definite points of space time. Quantum Theory on the other hand is not only probabilistic, but the Heisenberg Uncertainty Principle forbids the notion of a single point space time event: Four dimensional space time exists only as a classical approximation[3]. Inspite of this apparent contradiction, it is still possible to give a "local" and "causal" formulation of Quantum Theory evidenced by the fact that we deal with finite order differential equations (cf. ref.[2]).

This reconciliation notwithstanding the inherent contradiction between Quantum Theory and classical physics remains and has been articulated by for example, the EPR paradox. At the root is the issue of Quantum Mechanical acausality and nonlocality. The acausal nature of a Quantum Mechanical measurement, according to Einstein, violated what may be called "local realism"[4]: Individual elements of physical reality of a system are independent of measurements performed on any other system separated by a space like interval, that is not in direct causal interaction. On the other hand this is opposed to a feature of Quantum Theory, namely nonseparability[5], according to which two systems which interacted once cannot be assigned separate state vectors, whatever the spatial separation. This according to Schrodinger was "the characteristic of Quantum Mechanics".

In what follows we argue, in the light of some recent work, that the concepts of locality and causality are valid only at energies and momenta greater than those corresponding to time and space scales \hbar/mc^2 and \hbar/mc (the Compton wavelength) but breakdown as we approach smaller space time intervals. In the process we obtain a rationale for quantum nonseparability and argue that the "spooky" EPR-Aspect result may not be that "spooky" after all.

2 Quantum Mechanical Non Locality I

In Quantum Mechanics it is known that due to the Heisenberg Uncertainty Principle non local effects can exist within the Compton wavelength[6]. Indeed a recent model interprets the elementary particles as what may be called Quantum Mechanical Black Holes, described by a Kerr-Newman type metric with a horizon at the Compton wavelength and wherein a naked singular-

ity is shielded by the Zitterbewegung effects[7, 8, 9]. Within the Compton wavelength region we have non local effects characterised by a non Hermitian position operator. As pointed out there, physics with the conventional space time begins at scales greater than the Compton wavelength and at time scales greater than \hbar/mc^2 . The result is deduced for the idealized case of an isolated particle and could be modified when interactions are included, though it would remain true in spirit.

It is within this domain that concepts of locality and causality in the usual sense apply. Infact in Quantum Electrodynamics propagation with velocities less than, equal to or greater than the velocity of light c is allowed. However velocities less than or equal to c have overwhelmingly far greater probability[10] at larger distances on a microscopic scale. Indeed even in Classical Electrodynamics superluminal velocities appear within time intervals corresponding to the Compton wavelength (cf.ref.[1]).

All this could be understood from a slightly different perspective. It is well known that the Quantum Mechanical wave function which should provide as complete a description of the system, as is possible in principle, is complex because of the requirement of predictability by the correspondence principle[11] The description of the wave function as

$$\psi = Re^{iS}$$

leads to the hydrodynamical formulation[12, 13], from where as discussed in detail in ref.[8], we could get quantized vortices with circulation velocity that of light which can be identified with the Quantum Mechanical Black Hole referred to above.

That is, the requirement of predictability leads to complex Quantum Mechanical wave functions which leads to the above model in which superluminal velocities are relegated to a physically inaccessible region within the Compton wavelength. Luminal and subluminal velocities are encountered in the physically accessible region outside the Compton wavelength. This is symptomatic of the fact that in Quantum Mechanics, unlike in the classical theory, single space time points have no physical meaning. In the Quantum Field Theory of the Dirac equation the wave function at two space time points does not commute, and infact

$$\{\psi(x), \psi(x')\} = 0,$$

thus apparently violating microscopic causality which is expressed by

$$[\psi(x), \psi(x')] = 0$$

for space like intervals[2]. However the commutativity is restored at distances large compared to the Compton wavelength[14]. Moreover bilinear forms, which correspond to physical observations, do commute. Such bilinear forms correspond to densities with averages being taken over infinitesimal volumes. (Infact it is precisely such an averaging over a volume corresponding to the Compton wavelength that totally delinks negative energy components of the Dirac wave function corresponding to superluminal non local effects from the physical positive energy components (cf.ref.[8])).

Thus in all these cases we see that causal physics is restored at scales greater than the Compton wavelength.

3 Quantum Non Locality II

The EPR paradox, Bell's Theorem and the Aspect experiments have been much commented upon (cf.ref.[15]). It is the non local character of Quantum Mechanics, which was subsequently experimentally demonstrated to which Einstein could not reconcile himself. This non-locality is slightly different.

To analyse this feature we consider the following simplified experiment: Two identical particles which are initially together, possibly in a bound state, get separated and move along opposite directions. We do not consider the spin of the particles which could thus be taken to be spin less. If we measure the momentum of one particle, say P , then the momentum of the other particle Q , should have the same magnitude but with the opposite direction both in Classical and Quantum Theory. The "spooky" aspect of the Quantum Mechanical experiment is that the momentum of any particle, be it P or Q is determined only when an actual acausal, measurement is performed on that particle - in other words a measurement of the momentum of P should not throw any light on the momentum of Q , which latter can only be determined by a separate acausal measurement.

We observe that the so called paradox arises because of an application of the law of conservation of momentum which is valid both in Classical and Quantum Theory. Let us analyse this a little more closely.

This conservation law in Quantum Theory arises due to the assumption of

homogeneity of space[16]. Infact we define displacement (momentum) operators by considering an instantaneous infinitesimal shift of origin. The instantaneous nature of this spatial shift expresses the space homogeneity property: The displacement (or momentum) operators are independent of the particular space point. In a physical measurement, such an instantaneous shift corresponds to an infinite (superluminal) velocity: Infact the homogeneity of space is a non local property. In other words the conservation of momentum law as it stands expresses a non local property, which as in this case is compatible with a closed system (cf.ref.[16]). Alternatively this law is valid if the instantaneous displacement can also be considered to be an actual displacement in real time δt . This is the case when the Hamiltonian is not an explicit function of time t . This is a stationary or steady state case and it is only under these circumstances that the space and time displacement operators are on the same footing, that is we have a symmetry between space and time as in special relativity[17]. It is important to bear this in mind. Infact the symmetry between space and time has been overstated (cf.also ref.[3]). Our perception and description of the universe is "all space" at "one instant of time". Such a description is clearly non-local except for the steady state case.

The following circumstance provides further insight into the matter. In the light of the model discussed in[9] (cf. also[18]), wherein particles are created fluctuationally from the background Zero Point Field, the Compton wavelength l of the pion being the typical length and the pion itself a typical particle, we observe that the fundamental Quantum Mechanical uncertainty follows as a consequence, rather than as an apriori consideration, in the form of the well known equation,

$$l \sim R/\sqrt{N}$$

where R is the radius of the universe and $N \sim 10^{80}$ is the number of particles, typically pions.

It is worth mentioning that if there are N particles in a system and R is its dimension, then the typical uncertainty length l is given by the above relation (cf. also.ref.[19]). In the thermodynamic limit $N \rightarrow \infty$ this uncertainty length $\rightarrow 0$ and we are in the classical domain. In any case, as N is large the classical concept of conservation of momentum can be taken to hold.

From this point of view it appears that the very Quantum Mechanical behaviour which leads to non locality is a natural consequence of the holistic

nature of space itself with an inbuilt non-locality (cf.ref.[9] also). This will be further elaborated in the next section.

In the light of the above comments, the "spooky" or non local character is no longer so surprising, given the non-local character embodied in homogeneity (or conservation laws) and Quantum non-separability, which, we now argue, provides an underpinning for space.

4 Quantum Nonseparability and Metric

We have noted in the introduction that Schrodinger considered the nonseparability of two wave functions as the characteristic of Quantum Mechanics. This can be understood in terms of the "fluctuation model" (cf.ref.[9]): Particles are created from fluctuations of a background Zero Point Field trapped within the Compton wavelength, a model which as we will see briefly leads to a cosmology consistent with observation. In this model the various particles are interconnected or form a network by the background ZPF effects taking place within time intervals \hbar/mc^2 and corresponding to virtual photons of QED. Infact if two elementary particles, typically electrons, are separated by distance r , remembering that the spectral density of this field is given by[20], (cf.also ref.[9])

$$\rho(\omega)\alpha\omega^3$$

the two particles are connected by those quanta of the ZPF whose wave lengths are $\geq r$. So the force of (electromagnetic) interaction is given by,

$$\text{Force} \quad \alpha \int_r^\infty \omega^3 dR,$$

where

$$\omega\alpha\frac{1}{R},$$

R being a typical wavelength.

Finally,

$$\text{Force} \quad \alpha\frac{1}{R^2}$$

Thus in the idealised case of two stationary isolated particles, we have recovered the Coulomb law. This justifies Feynman's statement that action-

at-a-distance must have a close connection with field theory. (More precisely, as pointed out in reference[8], the Force field is given correctly by the Kerr-Newman metric. For a somewhat similar but simpler derivation of the Coulomb law cf.ref.[21]).

It is this property of interconnectivity of the particles which indeed defines a set of particles, which is an important starting point if we do not assume, as we should not, background space. The point is do we consider a background space as an apriori container of matter, or do we consider the material content of the universe itself defining space (cf.ref.[22] for a discussion). We adopt the latter viewpoint.

Starting now from the set (rather than manifold) of particles as above it is possible to define a metric. One way of doing this is by first defining the neighbourhood of an element as a subset of some universal set of particles, which contains the element a say, and atleast one other distinct element.

We now assume the following property: Given two distinct elements (or even subsets) a and b , there is a neighbourhood N_{a_1} such that a belongs to N_{a_1} , b does not belong to N_{a_1} and also given any N_{a_1} , there exists a neighbourhood $N_{a_{\frac{1}{2}}}$ such that $a \subset N_{a_{\frac{1}{2}}} \subset N_{a_1}$, that is there exists an infinite sequence of neighbourhoods between a and b . In other words we introduce topological closeness.

From here, as in the derivation of Urysohn's lemma[23], we could define a mapping f such that $f(a) = 0$ and $f(b) = 1$ and which takes on all intermediate values. We could now define a metric, $d(a, b) = |f(a) - f(b)|$. We could easily verify that this satisfies the properties of a metric.

The point is, that in the usual theory, we have apparently unconnected particles occupying the same background space. However, it is non-local linkages within time intervals of $\sim \hbar/mc^2$ that provide the underpinning for space itself (cf.ref.[9]).

A physical picture of the above consideration is obtained if we start with a set of n particles or subconstituents. There are 2^n subsets and as $n \rightarrow \infty$, $2^n \rightarrow C$ where C is the cardinal number of the real continuum. As each subset defines atleast one particle or subconstituent, we end up with a continuum in the above process even though we start off with a countable set.

Two points to be emphasized are: Firstly we had to define the set of particles, that is physically, we defined a rule which enables us to determine whether the particle belongs to the set or not. Secondly we arrived at a metric starting

from a larger set. This holistic aspect which we encountered in the previous section has been commented upon in[9] (cf.also ref.[24]), and is shown to be the reason why the pion mass is related to the Hubble constant, an otherwise inexplicable relation which Weinberg calls mysterious (cf.ref.[6]).

However two points need to be emphasized here:

Firstly, this "mysterious" relation,

$$m_\pi = \left(\frac{H\hbar^2}{Gc} \right)^{1/3}$$

actually follows from the theory in ref.[9].

Secondly, the related cosmological model[9, 25], apart from actually deducing the large number coincidences of cosmology, predicts an ever expanding, accelerating universe with decreasing density as has been observationally confirmed recently[26, 27].

5 Quantization of space time and Quanta

It was pointed out in the introduction that nonlocality arises in Classical Electrodynamics within time intervals of the order \hbar/mc^2 . This has lead to the concept of the chronon - a minimum unit interval τ_o of time of the same order[28]. This some what adhoc procedure eliminates in Classical theory the runaway solutions of Dirac's equations, but otherwise has no strong rationale. When we neglect τ_o^2 and higher orders we get back the usual classical equations. For time intervals smaller than the chronon, that is roughly less than 10^{-23} seconds, the motion can be random. But for larger time intervals, special relativity holds.

On the other hand this concept can be extended to the domain of Quantum Theory (cf.ref.[28] and also [29]). This leads to the fact that wave packets have a minimum spatial spread of the order of the Compton wavelength. Indeed even in the Classical Theory of the electron, if the minimum space spread or radius of the approximate spherical electron, $R \rightarrow 0$, we get saddled with the well known infinities.

The above two separate constraints on the minimum size of intervals in space and time emerge in a unified way in the model of a Quantum Mechanical Black Hole as discussed elsewhere (cf.references[8, 30]). Here as mentioned earlier the electron for example, is the Kerr-Newman type black hole,

bounded by the Compton wavelength, but with a naked singularity shielded by the unphysical Zitterbewegung effects (reminiscent of the random motion in the above chronon consideration), which disappear on averaging over intervals of the order of τ_0 . Further in this model as above the Compton wavelength gives a natural boundary for a wave packet: As we approach it we encounter unphysical negative energies corresponding to non Hermitian operators. The correct field of the electron, including the anomalous gyro magnetic ratio $g = 2$ emerges quite naturally. The whole point is as mentioned earlier, we cannot uncritically carry over classical concepts of space time to the micro domain, that is to the relatively high energy domain of Quantum Mechanics. This has been noted by a few scholars[31, 32]. (In Quantum Gravity too, such a granulation is recognised at the Planck scale[33]. This has not led to fruitful results though).

Special Relativity and related concepts of locality and causality are phenomena at space scales greater than the Compton wavelength and corresponding time intervals (the chronon): There is an ultimate quantization of space and time, a granularity, which is glossed over at our usual energy scales. The infinities we encounter in Classical and Quantum Theory are due to our extrapolating the usual theory in to a domain in which it is no longer valid, viz., a domain bounded by τ_0 and the Compton wavelength in which locality and causality no longer hold.

It is interesting to note that, given the Compton wavelength and τ_0 the velocity of light c appears as the limiting velocity our physical universe permits. If there were no such upper bound, then for a certain observer there would be no Quantum Mechanical Black Holes, that is no fermions or material content in the universe.

Infact in the Quantum Mechanical Black Hole model, as seen earlier the particles are created from the Zero Point Field within the Compton wavelength which is a cut off (cf.ref.[9]) - the spectral density of the ZPF itself being $\alpha\omega^3$, where ω is the frequency, a relation which is compatible with Special Relativity (cf.ref.[20]). Thus we see in this picture, a convergence of Special Relativity and Quantum Mechanics.

One could consider the minimum space and time intervals as being more fundamental than the quantization of energy. Indeed in elementary Classical Theory if the wave length has a discrete spectrum, then so does the frequency because their product equals the velocity of light. Alternatively the frequency is inversely proportional to the time period and hence will have a

discrete spectrum. This leads to Planck's law. The derivation is similar to the well known theory.

Infact let the energy be given by

$$E = g(\nu)$$

Then, f the average energy associated with each mode is given by,

$$f = \frac{\sum_{\nu} g(\nu) e^{-g(\nu)/kT}}{\sum_{\nu} e^{-g(\nu)/kT}}$$

Again, as in the usual theory[34], a comparison with Wien's functional relation, gives,

$$f = \nu F(\nu/kT),$$

whence,

$$E = g(\nu) \alpha \nu,$$

which is Planck's law.

Yet another way of looking at it is, as the momentum and frequency of the classical oscillator have discrete spectra so does the energy.

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